

Messiah College

Calculus I Placement Exam Topics and Review

The placement exam is designed to test a student's knowledge of material that is essential to the Calculus I course. A score of 60% or higher is required to pass the test. Students who score less than 60% are advised to take MATH 109-110 instead of MATH 111. The placement exam is to be taken with pencil and paper alone. Although calculators are generally allowed on chapter exams in our calculus courses, the use of a calculator or any computer-based computation is prohibited on the placement exam.

Topics covered on the exam, along with several sample questions for each topic, are listed below. The page numbers listed below indicate where the specified topic is found in the course textbook: *Calculus, 9th edition* by Larson, Hostetler, and Edwards.

Solutions to problems in the text are listed in the back of the book. Solutions to the example problems given below can be found in the file "Placement Exam Review Key", at the Math Department website.

Section P1: Graphs and Models

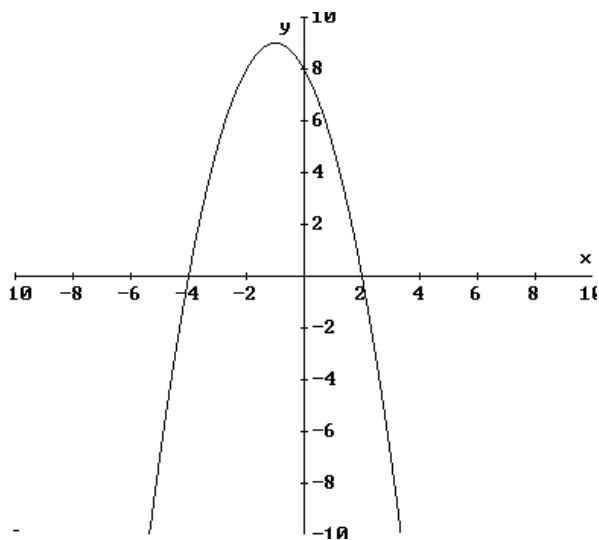
(pp.1-7 Exercises 1,2,3,4,5,9,21,41,43,45,47,63,65,69)

Also, know the following:

1. Equations and graphs of parabolas

ex. Find the vertex of the parabola $y = 2x^2 - x + 3$. Where does this parabola intersect the x-axis? y-axis?

ex. Find the equation of the parabola shown below.



2. Quadratic equations and the quadratic formula.

ex. Solve the following quadratic equations:

i. $2x^2 + 2x - 3 = 0$.

ii. $3x^2 - x = 3$

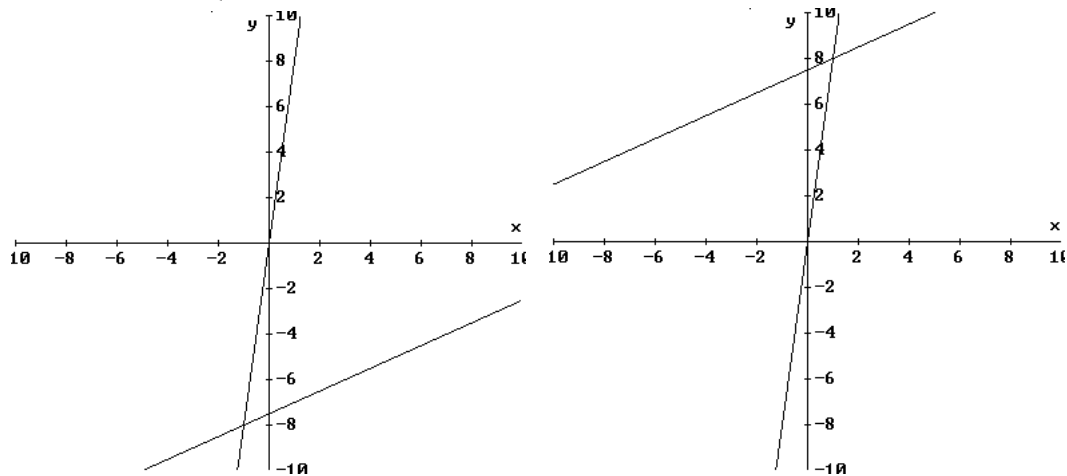
3. Find solution sets for simultaneous linear equations.

ex. Solve the following system of equations:

$$\begin{cases} y = 2x + 3 \\ y = -4x + 8 \end{cases}$$

ex. Which of the following graphs shows the system of equations

$$\begin{cases} \frac{x+y}{3} = y - 5x \\ 2y - x = 15 \end{cases} \text{ and its solution?}$$



4. Find solution sets for simultaneous equations in general.

ex. Solve the following systems of equations:

$$y = -5x + 6$$

$$y = x^2$$

ex. Find all intersections of the circle $x^2 + 10x + y^2 - 8y = 4$ with the line $x + y = 8$. Sketch the two curves and label the points of intersection.

Section P2: Linear Models and Rates of Change

(pp. 10-15 Exercises 1,3,9,15,25,29,33,37,41,45,47,53,61)

Also, know the following:

1. linear equations including perpendicular and parallel lines.
 - ex. Identify the slope and the intercepts for the equation $3x + 4y = 12$. Sketch the graph of this equation.
 - ex. Find the equation of the line passing through the point $(-1,3)$ and parallel to the line $3x + 4y = 12$.
 - ex. Find the equation of the line passing through the points $(1,-1)$ and $(1,1000)$.

Section P3: Functions and Their Graphs

(pp. 19-26 Exercises 3,7,9,11,17,21,23,27,35,37,45)

Also, know the following:

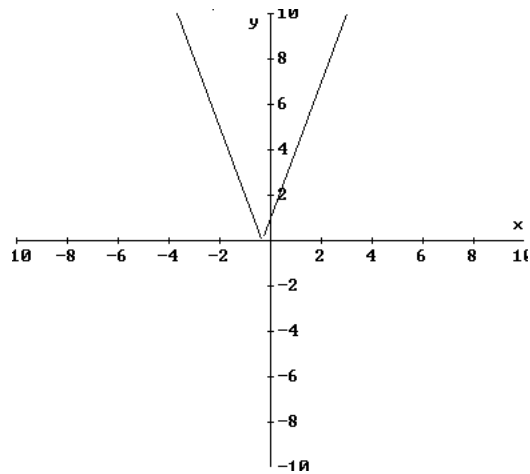
1. Domain of a function:
 - ex. Find the domain of $\frac{1}{\sqrt{x^2 - 16}}$
2. Functional notation and equations as functions:
 - ex. For the function $f(x) = \frac{1}{\sqrt{x^2 - 16}}$, find:
 - a) $f(5)$
 - b) $f(a)$
 - c) $f(a+3)$
3. Use translations and shifting to describe the graphs of functions.
 - ex. Use the graph of $f(x) = x^2$ to sketch the following graphs:
 - a) $f(x+2) = (x+2)^2$
 - b) $f(x)+2 = x^2+2$
 - c) $2f(x)-3 = 2x^2-3$

Real Numbers and the Real Line

1. Know the definition of absolute value and solve equalities and inequalities involving absolute value:

ex. $|2x + 5| = 3$

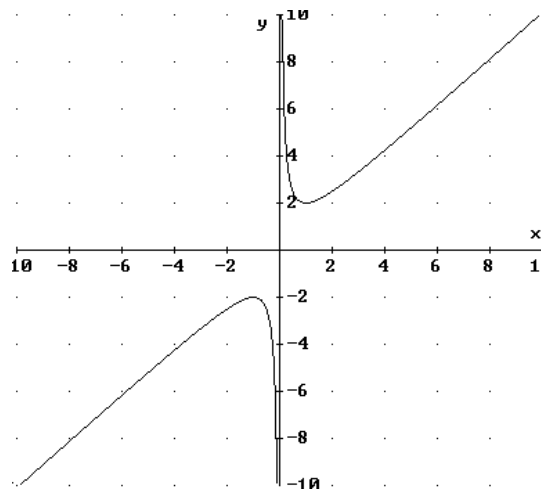
Shade the portion of the x-axis that corresponds to the solution of $|3x + 1| > 5$, given the graph of $y = |3x + 1|$ below.



2. Solve inequalities in general:
ex. Solve $x^2 + 1 > 4x$

Shade the portion of the x-axis that corresponds to the solution of $x + \frac{1}{x} \geq 1$,

given the graph of $y = x + \frac{1}{x}$ below.



- ex. Solve $x^2 + x - 2 \leq 0$

The Cartesian Plane

1. Distance and midpoint formulas

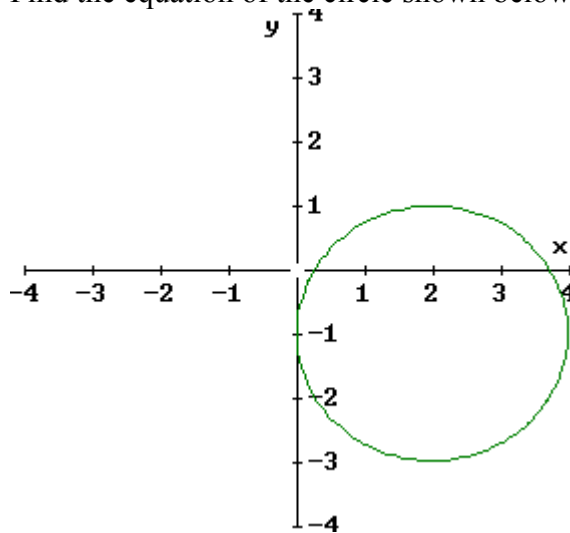
ex. Find the distance between (2,3) and (-1,4).

ex. Do the following points form a right triangle: (4,0), (2,1), (-1,-5)?

2. Equation of circles and completing the square:

ex. Find the radius and center of the circle $x^2 - 4x + y^2 + 2y = -1$. Sketch the graph.

ex. Find the equation of the circle shown below.



Review of Trigonometric Functions

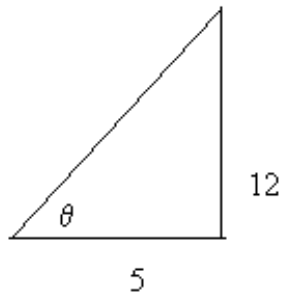
1. the amplitude and period of functions of the form $y = A \sin Bx$ or $y = A \cos Bx$.

ex. Sketch the graph of $y = 3 \sin 2\pi x$. What is the amplitude and period of this function.

ex. Sketch the graph of $y = 3 + 5 \cos 12x$. State the amplitude and period of this function.

ex. If $\tan \theta = \sqrt{3}$ and $0 \leq \theta \leq \frac{\pi}{2}$, what is θ in radians?

ex. What are $\sin \theta$, $\cos \theta$, and $\sec \theta$?



2. simple trigonometric equations.

ex. Solve the following equations:

i. $\sin \theta = \cos \theta$.

ii. $\sin \theta \cos \theta = 5 \cos \theta$.

Other Problems

1. Simplify the following:

ex. $\frac{(3y^3)(2y^2)^2}{(y^4)^3}$

ex. $\left(\frac{4a^2b}{a^3b^2}\right)\left(\frac{5a^2b}{2b^4}\right)$

ex. $(5x^2y^{-2})(4x^{-5}y^4)$

ex. $\left(\frac{-8x^3}{y^{-6}}\right)^{2/3}$

2. Perform the indicated operation and simplify:

ex. $(3u-1)(u-4) + 7u(u+1)$

ex. $\frac{5a^2 + 12a + 4}{a^4 - 16} \div \frac{25a^2 + 20a + 4}{a^2 - 2a}$

ex. $\frac{4x}{3x-4} + \frac{8}{3x^2-4x} + \frac{2}{x}$

ex. $\frac{9x^2-4}{3x^2-5x+2} \cdot \frac{9x^4-6x^3+4x^2}{27x^4+8x}$

ex. $\frac{\frac{x}{y^2} - \frac{y}{x^2}}{\frac{1}{y^2} - \frac{1}{x^2}}$

3. Factor polynomials.

ex. Factor the following:

i. $x^4 - 16$

ii. $2x^4 - 5x^3 - 3x^2$

iii. $3x^3 + 3x^2 - 27x - 27$